Math 125

Spring 2021

Lecture 12

$y = ax^2 + bx + c$
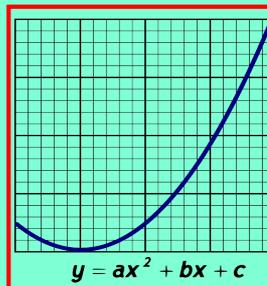
Is $(-2, -6, 3)$ a Solution of

$$\begin{cases} 5x - y + 3z = -7 \\ 3x + 4y - z = 5 \\ 9x + 5y + 7z = 1 \end{cases} \ ?$$

$5x - y + 3z = -7$

$5(-2) - (-6) + 3(3) = -7$

$-10 + 6 + 9 = -7$

$-10 + 15 = -7$

$\boxed{5 = -7}$ False

Not a Solution.

Solve

$$\begin{cases} 5x - y + 3z = -7 \\ 3\begin{cases} 3x + 4y - z = 5 \end{cases} \end{cases}$$

$\overline{}$

$14x + 11y = 8$

$7\begin{cases} 3x + 4y - z = 5 \\ 9x + 5y + 7z = 1 \end{cases}$

$\overline{}$

$30x + 33y = 36$

Divide by 3 ⟹ $10x + 11y = 12$

$$\begin{cases} 14x + 11y = 8 \\ - \ \ 10x + 11y = 12 \end{cases}$$

$\overline{}$

$4x \quad = -4$

$\boxed{x = -1}$

$14(-1) + 11y = 8$

$11y = 8 + 14$

$11y = 22$

$\boxed{y = 2}$

$5x - y + 3z = -7$

$5(-1) - 2 + 3z = -7$

$-7 + 3z = -7$

$\boxed{z = 0}$

Final Ans $(-1, 2, 0)$

System is Consistent. Eqns are indep.

Solve

$$\begin{cases} x + y + 4z = -1 \\ 3x + y - 4z = 3 \\ -4x - y + 8z = -2 \end{cases}$$

$$-1\begin{cases} x + y + 4z = -1 \\ 3x + y - 4z = 3 \end{cases}$$

$$\overline{\quad 2x \qquad\quad -8z = 4}$$

Divide by 2

$$\boxed{x - 4z = 2}$$

$$\begin{cases} x + y + 4z = -1 \\ -4x - y + 8z = -2 \end{cases}$$

$$\overline{-3x \qquad\quad +12z = -3}$$

Divide by -3

$$\boxed{x - 4z = 1}$$

$$\begin{cases} x - 4z = 2 \\ - \begin{cases} x - 4z = 1 \end{cases} \end{cases}$$

$$\overline{\qquad 0 = 1 \text{ False}}$$

No Solution

System is inConsistent

Independent eqns.

Solve

$$\begin{cases} x + 2y + 4z = 3 \\ \quad\;\; y + 3z = 5 \\ x \qquad\quad -2z = -7 \end{cases} \Rightarrow \begin{cases} x + 2y + 4z = 3 \\ -2 \begin{cases} \quad\; y + 3z = 5 \end{cases} \end{cases}$$

$$\overline{\qquad x \qquad\quad -2z = -7}$$

$$\begin{cases} x - 2z = -7 \\ -1 \begin{cases} x - 2z = -7 \end{cases} \end{cases}$$

$$\overline{\qquad 0 = 0}$$

True

$\rightarrow$ Infinte # of Solutions

Equations are dependent.

System is Consistent.

System of non-linear equations

$$\begin{cases} x = -7y - 50 \\ x^2 + y^2 = 100 \end{cases}$$

Use Subs. method

$(-7y-50)^2 + y^2 = 100$

$(-7y-50)(-7y-50) + y^2 = 100$

$49y^2 + 350y + 350y + 2500 + y^2 - 100 = 0$

$50y^2 + 700y + 2400 = 0$

Divide by 10

$5y^2 + 70y + 240 = 0$

Divide by 5

$y^2 + 14y + 48 = 0$

$(y+6)(y+8) = 0$

$y = -6 \qquad y = -8$

$y = -6$

$x = -7y - 50$

$= -7(-6) - 50$

$= 42 - 50 \Rightarrow \boxed{x = -8}$

$y = -8$

$x = -7(-8) - 50 = 56 - 50 = 6$

$\boxed{x = 6}$

$(-8, -6), (6, -8)$

Solve

$$\begin{cases} 2x + y = 5 \\ x^2 + y^2 = 50 \end{cases}$$

Hint: Isolate one Variable, and Use Subs. method.

$$\begin{cases} y = 5 - 2x \\ x^2 + (5-2x)^2 = 50 \end{cases}$$

$x^2 + 25 - 20x + 4x^2 - 50 = 0$

$5x^2 - 20x - 25 = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x - 5 = 0 \qquad x + 1 = 0$

$x = 5 \qquad x = -1$

$y = 5 - 2(5) \qquad y = 5 - 2(-1)$

$\boxed{y = -5} \qquad \boxed{y = 7}$

Final Ans.

$\Rightarrow \boxed{(5, -5), (-1, 7)}$

Solve by elimination

$$-2 \begin{cases} 2x^2 + y^2 = 17 \\ x^2 + 2y^2 = 22 \end{cases} \Rightarrow \begin{cases} -4x^2 - 2y^2 = -34 \\ x^2 + 2y^2 = 22 \end{cases}$$

$$-3x^2 = -12$$

$$2x^2 + y^2 = 17$$
$$2(4) + y^2 = 17$$
$$y^2 = 9 \quad \boxed{y = \pm 3}$$

$$x^2 = 4$$
$$\boxed{x = \pm 2}$$

Final Ans:
$$\boxed{(2,3), (2,-3), (-2,3), (-2,-3)}$$

Solve by elimination

$$2 \begin{cases} x^2 + y^2 = 17 \\ x^2 - 2y^2 = -31 \end{cases}$$

$$3x^2 = 3$$

$$x^2 = 1$$
$$x = \pm 1$$

$$x^2 + y^2 = 17$$
$$1 + y^2 = 17$$
$$y^2 = 16$$
$$\boxed{y = \pm 4}$$

$$\boxed{(1,4)\,(1,-4), (-1,4), (-1,-4)}$$

Final Ans.